Supplementary Material

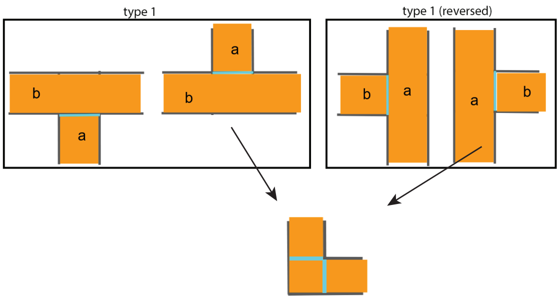

Figure 1: *Top: Four cases of connection type 1 for a pair of parts, viewed from the side so that both parts' sheet normals are parallel to the page. Bottom: one corner configuration determined from the orientations of both candidate type 1 connections. Interface surfaces are shown in cyan.*

1. Cut Region Approximation

The function from which we extract isocontours to initially approximate the cut shape in Section 4.2.2 is

$$f(\mathbf{x}) = \sum_{\mathbf{q}} \exp(-\|\mathbf{q}-\mathbf{x}\|^2/(2\sigma)) \tag{1}$$

where $\mathbf{q}$ are the 2D points, and σ is $(D/400)$. We extract a level set $\{x|f(x) = 1/e\}$ using the Marching Squares algorithm with a grid length of 4σ.

2. Inferring Connection Types

To detect whether part b approximately terminates at part a's sheet plane, thus forming a T-junction necessary for considering type 1 connections, we use part a's sheet normal $\mathbf{n}_a$, and a's sheet plane offsets o_{min} and o_{max} to compute the projected offsets of all points in part b: $z_{\text{min}} = \min_{\mathbf{p}\in P_b}(\mathbf{p}\cdot\mathbf{n}_a)$ and $z_{\text{max}} = \max_{\mathbf{p}\in P_b}(\mathbf{p}\cdot\mathbf{n}_a)$. If

$$z_{\text{min}} < o_{\text{min}} - \tau_c \tag{2}$$

and

$$z_{\text{max}} > o_{\text{max}} + \tau_c \tag{3}$$

are both true, part b is not confined to either side of part a's sheet, preventing connection type 1. As long as only one is true, the connection is allowed; the interface plane offset is o_{min} if (2) holds, and o_{max} if (3) holds. Type 2 connections occur when a type 1 connection is valid for both orderings of parts a and b.

Consequently, there are actually 4 discrete configurations for a type 1 connection between two parts, as shown in Figure 1: For the connection and its reverse (where a and b are swapped), the connection may involve contact with one of two sides of the sheet plane (whether the interface plane offset is o_{min} or o_{max}). For type 2 connections, the positions of these contacts for both the the type 1 connection and its reverse are used to determine the corner configuration, as shown in the bottom of Figure 1; knowing where the potential contact surfaces lie is crucial to knowing where in the images to look for seams.

We assume corners (type 2 connections) are right angles. Though it would not be difficult to allow them to vary, our choice of two "natural" corner configurations requiring only orthogonal cuts no longer makes sense; bevel cuts will be needed no matter what, so potentially more complex joints would need to be detected. Furthermore, non-orthogonal corners are uncommon.

3. Curve Fitting

The dynamic programming curve fitting algorithm can infer the optimal set of nodes from the input point set, with the caveat that it requires a starting point from which the optimal sub-ranges belonging to separate curves are determined. This starting point is necessarily a node, since it is the start of the first such range returned by the algorithm. A first instinct might be to choose a starting point that looks like it should be a corner; however, if the input shape has no true sharp corners, the exact tangent behavior at the start of the loop will depend on the behavior at the end of the loop, which violates the sequential order in which we find the curves.

Instead, we do the opposite: We look for a starting node in a region that is as flat as possible (which we determine using the curvature of a Bézier curve fit to a neighborhood of points centered at the query point). Such a region can be assumed to always exist for well-behaved inputs approximating continuous shapes, and allows the tangents at the loop boundaries to be assumed to be smooth. The downside is that the starting node often bisects a region that could be better described with a single curve or line segment. We therefore filter out the extra node whenever possible by merging collinear line segments (the most likely case).

In practice, it is very inefficient to consider every possible subrange of points as a candidate curve. The space and time complexity of the dynamic programming algorithm is quadratic in the number

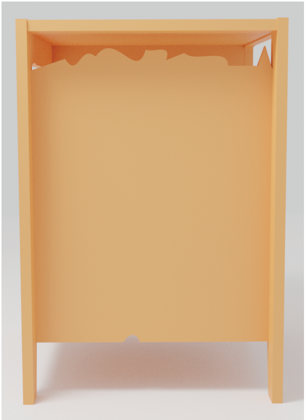

Figure 2: *Without the image segmentation step, deficiencies in the input point cloud persist in the final result.*

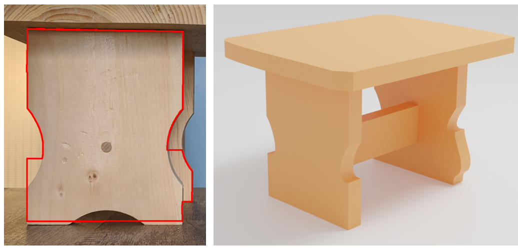

Figure 3: *Results from using only one image per part in the segmentation phase. Left: segmentation result for one part, with the segmented shape superimposed in red on the corresponding image. Right: The full result.*

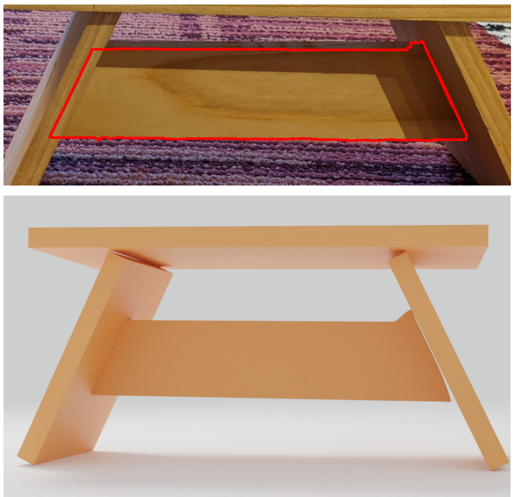

Figure 4: *In the presence of some segmentation artifacts (top), unconstrained curve fitting produces an incorrect result (bottom). Note that the supposed contact between the central piece and the right leg is curved.*

of candidate nodes, and this number is potentially very large when the input is a dense bitmap mask boundary. So given a maximum number of candidate nodes K, we find the K input points with the highest curvature and mark them as candidates, since such corners are likely to mark the boundaries between separate curves.

The full recurrence relation for our energy function E_{ijk} is

$$E_{ijk} = \min_{j',k'}(E_{ij'k'} + e_{j'jk'k}), i < j' < j \tag{4}$$

where i and j are the start and end points of the considered range of points, and k is the type of curve fit to the range ending at j (so E_{ijk} is the energy of a sequence of curves whose last curve has type k). Note that the sub-range energy $e_{j'jk'k}$ depends not only on the starting and end node indices, but also the *type* of the previous and current curve. Because k also defines whether a Bézier curve should use the fixed (precomputed) tangent at its last endpoint, this allows us to define the rules, detailed in Section 4.4.4, governing the behavior of neighboring curves, including the angles between their tangents where they meet.

4. Evaluating Design Choices

To gauge the necessity of some of the stages of our pipeline, we discuss how disabling or simplifying them impacts the quality of results.

Joint Image-Based Segmentation Figure 2 shows the result of skipping image segmentation, and directly applying curve fitting to the point set boundaries from Section 4.2.2. Because regions of the model with less visibility, such as cavities and undersides, are often missing from the point cloud, the final result contains gaps. Compared to the full pipeline, we lack the ability to expand part shapes into regions of similar texture, which is a method to close gaps such as this.

Using Multiple Views in Segmentation We also show results from using only a single image per part in the segmentation phase in Figure 3. Without multiple views to disambiguate foreground and background, similarly-colored surfaces become erroneously associated with the part shape, leading to an artifact-laden result. In general, material and lighting conditions can make discerning part shapes from certain views difficult; averaging the segmentation energy over multiple reprojected views exploits the view-dependence of pixels not belonging to the part, since similarly-colored background surfaces are less likely to interfere with the same pixels in every view.

Constrained Curve Fitting The constraints in the curve fitting stage straighten curves in the vicinity of connection contacts, effectively flattening nearby artifacts arising from the segmentation stage (which occur due to the poor visibility at some junctions). In our tilted stool example, one such artifact occurs due to heavy shadowing (see the top of Figure 4), but our constrained curve fitting gives a clean result (see Figure 8 in the main paper). The bottom of Figure 4 shows the result without considering constraints in the curve fitting stage. The artifact persists; furthermore, the contacts between all parts are imperfect. Amending the contacts purely in a post-process gives rise to new challenges, for without enforcing straight segments in the vicinity of contacts, there may not be a single part of the boundary curve that can be "snapped" to the surface. For example, the cut edge on the right of the central part has been chosen as part of a longer, continuous curve.