A Weights derivation for cases in Figure 6

A.1 Review of previously-covered material in paper

Recall that in order to render crack-free isosurfaces, we must reconstruct the value of the field at the corners of our "octants". For an octant that lies on the coarser side of a level boundary, 20 cases exist. (Figure 6 in the paper, and Figure 1 in this appendix.) Note that points colored in black correspond to the centers of cells on the coarse side, whereas points colored in green correspond to cell centers on the fine side.

A.2 Overview

Our goal is to find the value of the field Φ at point T, given that the values at vertices 0 through 7 are known a priori as Φ_1 through Φ_7. (T, illustrated in red in Figure 1, is the position of the corner of the octant that we are interested in. This point is denoted as O_{xyz} in the paper, but we use T here for brevity.) Denoting the value of Φ at T as Φ_T, we can express Φ_T as the weighted sum of the known field values Φ_i where c_i are scalar weights:

$$\Phi_T = \sum_{i=0}^{7} c_i \Phi_i \tag{1}$$

In this appendix, we will show how the weights c_i are found for each of the cases (a) through (t).

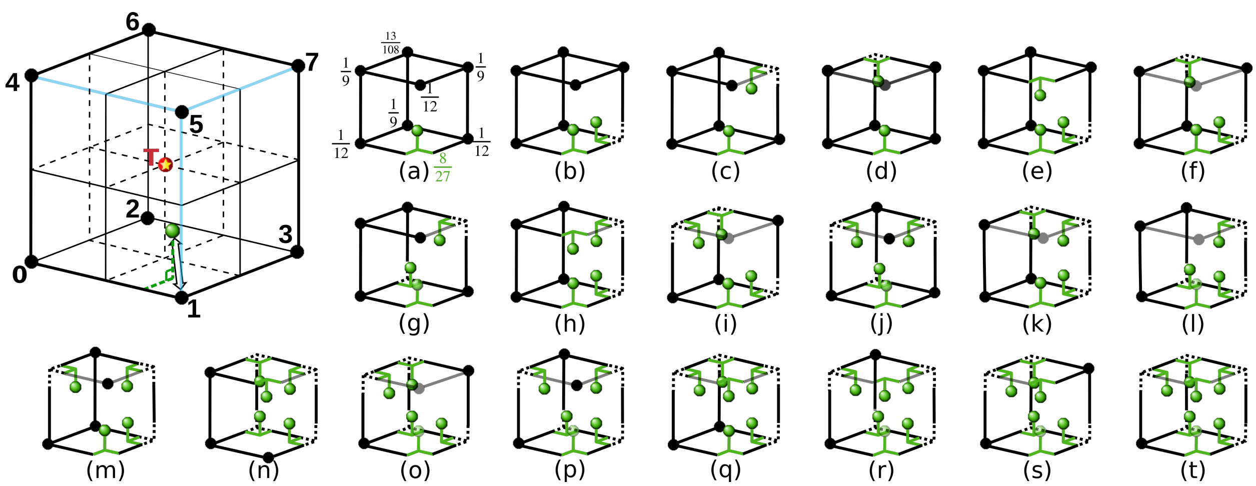

Figure 1: Alternate version of Figure 6 in the main part of the paper. Figure 6 is not drawn to scale, however this figure is drawn to scale using an orthographic projection. Some edges are transparent (gray) in order to clearly show the geometry behind the edges. The edge between vertex 1 and 5 often occludes the geometry behind it, so it has been omitted from the illustrations of cases (a) through (t). For reference, weights are shown for case (a). Please see Table 5 for an exhaustive list of weights.

A.3 Case (a)

Let us start with a case where one vertex belongs to a finer level cell (vertex 1, in green), and the other seven vertices belong to coarser level cells (0, 2, 3, 4, 5, 6, and 7). See Figure 2 for reference.

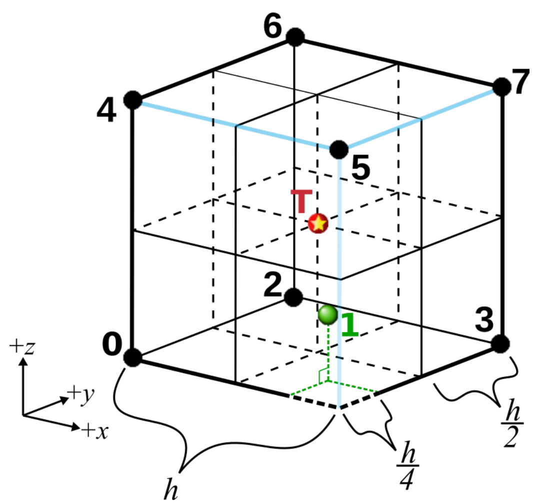

Figure 2: Diagram for case (a). See Table 1 for vertex positions.

For convenience, let us choose a coordinate system that **puts T at the origin**. The orientation of the coordinate axes is shown in Figure 2. Assuming this coordinate system, the coordinates of vertices 0 through 7 are given in Table 1. With these vertex positions, we can build our multivariate Vandermonde [?] matrix

Vertex	Position
0	$^1/_2(-h,-h,-h)$
1	$^1/_4(h,-h,-h)$
2	$^1/_2(-h,h,-h)$
3	$^1/_2(h,h,-h)$
4	$^1/_2(-h,-h,h)$
5	$^1/_2(h,-h,h)$
6	$^1/_2(-h,h,h)$
7	$^1/_2(h,h,h)$

Table 1: Vertex positions for case (a). See Figure 2 for an illustration.

V, which is used in Equations 7, 8 and 9 in the main paper. V is given as follows, where x_i, y_i, and z_i are the x, y, and z coordinates of vertex i:

$$V = \begin{bmatrix} 1 & x_0 & y_0 & z_0 & x_0y_0 & x_0z_0 & y_0z_0 & x_0y_0z_0 \\ 1 & x_1 & y_1 & z_1 & x_1y_1 & x_1z_1 & y_1z_1 & x_1y_1z_1 \\ 1 & x_2 & y_2 & z_2 & x_2y_2 & x_2z_2 & y_2z_2 & x_2y_2z_2 \\ 1 & x_3 & y_3 & z_3 & x_3y_3 & x_3z_3 & y_3z_3 & x_3y_3z_3 \\ 1 & x_4 & y_4 & z_4 & x_4y_4 & x_4z_4 & y_4z_4 & x_4y_4z_4 \\ 1 & x_5 & y_5 & z_5 & x_5y_5 & x_5z_5 & y_5z_5 & x_5y_5z_5 \\ 1 & x_6 & y_6 & z_6 & x_6y_6 & x_6z_6 & y_6z_6 & x_6y_6z_6 \\ 1 & x_7 & y_7 & z_7 & x_7y_7 & x_7z_7 & y_7z_7 & x_7y_7z_7 \end{bmatrix} \tag{2}$$

Recall from Equation 9 in the main paper that we can find the vector of weights $\mathbf{c} = (c_0, c_1, \ldots, c_7)$ by solving the linear system $V^T\mathbf{c} = \mathbf{e}_1$, where $\mathbf{e}_1$ denotes the canonical basis vector $(1, 0, \ldots, 0)^T$. This linear system is written explicitly below:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ y_0 & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 \\ z_0 & z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & z_7 \\ x_0y_0 & x_1y_1 & x_2y_2 & x_3y_3 & x_4y_4 & x_5y_5 & x_6y_6 & x_7y_7 \\ x_0z_0 & x_1z_1 & x_2z_2 & x_3z_3 & x_4z_4 & x_5z_5 & x_6z_6 & x_7z_7 \\ y_0z_0 & y_1z_1 & y_2z_2 & y_3z_3 & y_4z_4 & y_5z_5 & y_6z_6 & y_7z_7 \\ x_0y_0z_0 & x_1y1z_1 & x_2y_2z_2 & x_3y_3z_3 & x_4y_4z_4 & x_5y_5z_5 & x_6y_6z_6 & x_7y_7z_7 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

Substituting the vertex positions given in Table 1, we obtain V for case (a):

$$V = \begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix} \tag{4}$$

Using V as given above, and solving the linear system given in Equation 9 in the main paper (Equation 3 in this appendix), we obtain our weights c_0 through c_7 below (Equation 5). Importantly, due to cancellation, the final result is *independent* of h.

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} \frac{1}{12} \\ \frac{8}{27} \\ \frac{1}{9} \\ \frac{1}{12} \\ \frac{1}{9} \\ \frac{1}{12} \\ \frac{13}{108} \\ \frac{1}{9} \end{bmatrix} \tag{5}$$

Now, assuming that case (a) holds, if the field values Φ_0, Φ_1, …, Φ_7 are known, we can use the above results with Equation 1 to reconstruct the value of Φ_T:

$$\Phi_T = \frac{1}{12}\Phi_0 + \frac{8}{27}\Phi_1 + \frac{1}{9}\Phi_2 + \frac{1}{12}\Phi_3 + \frac{1}{9}\Phi_4 + \frac{1}{12}\Phi_5 + \frac{13}{108}\Phi_6 + \frac{1}{9}\Phi_7 \tag{6}$$

A.4 Case (b)

In case (b), two vertices (instead of one) correspond to the centers of finer level cells. These two vertices (1 and 3) are illustrated in green.

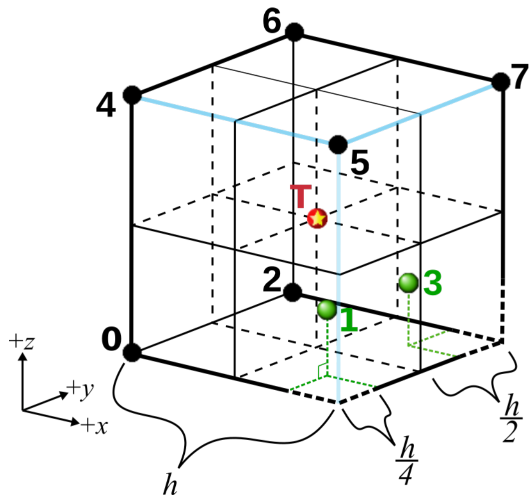

Figure 3: Diagram for case (b). See Table 2 for vertex positions.

Vertex positions for case (b) are given in Table 2. Note that now both vertex 1 and vertex 3 are closer to the origin. (Recall that, by construction, the origin coincides with point T.)

Vertex	Position
0	$^1/_2(-h,-h,-h)$
1	$^1/_4(h,-h,-h)$
2	$^1/_2(-h,h,-h)$
3	$^1/_4(h,h,-h)$
4	$^1/_2(-h,-h,h)$
5	$^1/_2(h,-h,h)$
6	$^1/_2(-h,h,h)$
7	$^1/_2(h,h,h)$

Table 2: Vertex positions for case (b). See Figure 3 for an illustration.

Substituting the vertex positions given in Table 2, into Equation 2, we obtain V for case (b):

$$V = \begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix} \tag{7}$$

Solving the linear system given in Equation 3, we obtain the following weights for case (b):

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} \frac{1}{12} \\ \frac{2}{9} \\ \frac{1}{12} \\ \frac{2}{9} \\ \frac{1}{9} \\ \frac{1}{12} \\ \frac{1}{9} \\ \frac{1}{12} \end{bmatrix} \tag{8}$$

A.5 Case (c)

Case (c) is like case (b), but with slightly different geometry. We will produce different weights since the relative positions of the vertices that belong to the finer level cell centers are different.

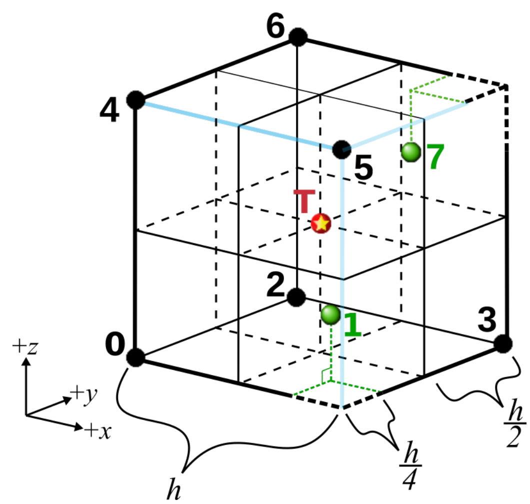

Figure 4: Diagram for case (c). See Table 3 for vertex positions.

Vertex	Position
0	$1/2(-h,-h,-h)$
1	$1/4(h,-h,-h)$
2	$1/2(-h,h,-h)$
3	$1/2(h,h,-h)$
4	$1/2(-h,-h,h)$
5	$1/2(h,-h,h)$
6	$1/2(-h,h,h)$
7	$1/4(h,h,h)$

Table 3: Vertex positions for case (c). See Figure 4 for an illustration.

We find the Vandermonde matrix V for Case (c) as we have done for the previous cases. V is given as follows:

$$V = \begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix} \tag{9}$$

Given V, we solve for the weights like we did before, and obtain:

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} \frac{1}{12} \\ \frac{4}{15} \\ \frac{1}{10} \\ \frac{1}{20} \\ \frac{1}{10} \\ \frac{1}{20} \\ \frac{1}{12} \\ \frac{4}{15} \end{bmatrix} \tag{10}$$

A.6 Cases (d) to (t)

Above, we derive weights for cases (a), (b), and (c). All other cases are derived analogously. For brevity, a detailed derivation will be omitted for cases (d) to (t). Instead, see Table 4 for the vertex positions used for each case, and Table 5 for the matrix V and final weights used in each case.

Case	Position of vertex							
	0	1	2	3	4	5	6	7
a	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/2(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/2(h,h,h)$
b	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/2(h,h,h)$
c	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/2(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
d	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/2(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/4(-h,h,h)$	$1/2(h,h,h)$
e	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/2(-h,-h,h)$	$1/4(h,-h,h)$	$1/2(-h,h,h)$	$1/2(h,h,h)$
f	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/4(-h,h,h)$	$1/2(h,h,h)$
g	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/2(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
h	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/2(-h,-h,h)$	$1/4(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
i	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/2(h,-h,h)$	$1/4(-h,h,h)$	$1/2(h,h,h)$
j	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/2(h,h,-h)$	$1/4(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
k	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/4(-h,h,h)$	$1/4(h,h,h)$
l	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/4(h,h,-h)$	$1/2(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
m	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
n	$1/2(-h,-h,-h)$	$1/2(h,-h,-h)$	$1/4(-h,h,-h)$	$1/4(h,h,-h)$	$1/2(-h,-h,h)$	$1/4(h,-h,h)$	$1/4(-h,h,h)$	$1/4(h,h,h)$
o	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/2(h,-h,h)$	$1/4(-h,h,h)$	$1/2(h,h,h)$
p	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/2(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
q	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/2(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/4(h,-h,h)$	$1/4(-h,h,h)$	$1/4(h,h,h)$
r	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/4(h,-h,h)$	$1/2(-h,h,h)$	$1/4(h,h,h)$
s	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/4(h,-h,h)$	$1/4(-h,h,h)$	$1/2(h,h,h)$
t	$1/2(-h,-h,-h)$	$1/4(h,-h,-h)$	$1/4(-h,h,-h)$	$1/4(h,h,-h)$	$1/4(-h,-h,h)$	$1/4(h,-h,h)$	$1/4(-h,h,h)$	$1/4(h,h,h)$

Table 4: Vertex positions for all cases in Figure 1. For instance, the second column (labeled "0") gives the position of vertex 0. The third column (labeled "1") gives the position of vertex 1, etc.

Case	Vandermonde matrix V	Weights vector $[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7]^T$
(a)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{1}{12}, \frac{8}{27}, \frac{1}{9}, \frac{1}{12}, \frac{1}{9}, \frac{1}{12}, \frac{13}{108}, \frac{1}{9}\right]^T$
(b)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{1}{12}, \frac{2}{9}, \frac{1}{12}, \frac{2}{9}, \frac{1}{9}, \frac{1}{12}, \frac{1}{9}, \frac{1}{12}\right]^T$

(c)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{12},\ \frac{4}{15},\ \frac{1}{10},\ \frac{1}{20},\ \frac{1}{10},\ \frac{1}{20},\ \frac{1}{12},\ \frac{4}{15}\right]^T$
(d)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{1}{14},\ \frac{2}{7},\ \frac{1}{14},\ \frac{1}{14},\ \frac{1}{14},\ \frac{1}{14},\ \frac{2}{7},\ \frac{1}{14}\right]^T$
(e)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{1}{12},\ \frac{4}{27},\ \frac{1}{12},\ \frac{2}{9},\ \frac{1}{12},\ \frac{2}{9},\ \frac{11}{108},\ \frac{1}{18}\right]^T$
(f)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{13}{180},\ \frac{2}{9},\ \frac{1}{20},\ \frac{26}{135},\ \frac{2}{27},\ \frac{13}{180},\ \frac{4}{15},\ \frac{1}{20}\right]^T$
(g)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{7}{132},\ \frac{8}{33},\ \frac{8}{33},\ \frac{1}{44},\ \frac{1}{11},\ \frac{7}{132},\ \frac{7}{132},\ \frac{8}{33}\right]^T$

(h)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{12},\ \frac{1}{6},\ \frac{1}{12},\ \frac{1}{6},\ \frac{1}{12},\ \frac{1}{6},\ \frac{1}{12},\ \frac{1}{6}\right]^T$
(i)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{1}{20},\ \frac{1}{5},\ \frac{1}{20},\ \frac{1}{5},\ \frac{1}{5},\ \frac{1}{20},\ \frac{1}{5},\ \frac{1}{20}\right]^T$
(j)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{36},\ \frac{2}{9},\ \frac{2}{9},\ \frac{1}{36},\ \frac{2}{9},\ \frac{1}{36},\ \frac{1}{36},\ \frac{2}{9}\right]^T$
(k)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{2}{27},\ \frac{2}{9},\ \frac{1}{18},\ \frac{4}{27},\ \frac{2}{27},\ \frac{1}{18},\ \frac{2}{9},\ \frac{4}{27}\right]^T$
(l)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{18},\ \frac{2}{9},\ \frac{2}{9},\ \frac{2}{27},\ \frac{5}{54},\ \frac{1}{18},\ \frac{1}{18},\ \frac{2}{9}\right]^T$

(m)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{14}{267}, \frac{52}{267}, \frac{13}{178}, \frac{14}{89}, \frac{22}{89}, \frac{5}{178}, \frac{14}{267}, \frac{52}{267}\right]^T$
(n)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{2}{27}, \frac{1}{18}, \frac{1}{6}, \frac{1}{6}, \frac{1}{18}, \frac{2}{9}, \frac{1}{6}, \frac{5}{54}\right]^T$
(o)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{1}{30}, \frac{1}{5}, \frac{4}{27}, \frac{7}{45}, \frac{1}{5}, \frac{7}{135}, \frac{7}{45}, \frac{1}{18}\right]^T$
(p)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{2} & -\frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{33}, \frac{13}{66}, \frac{13}{66}, \frac{1}{11}, \frac{5}{22}, \frac{1}{33}, \frac{1}{33}, \frac{13}{66}\right]^T$
(q)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & -\frac{h}{2} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{18}, \frac{1}{6}, \frac{1}{18}, \frac{1}{6}, \frac{1}{6}, \frac{1}{9}, \frac{1}{6}, \frac{1}{9}\right]^T$

(r)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{2} & \frac{h}{2} & \frac{h}{2} & -\frac{h^2}{4} & -\frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{30},\ \frac{1}{6},\ \frac{1}{5},\ \frac{1}{10},\ \frac{1}{5},\ \frac{1}{10},\ \frac{1}{30},\ \frac{1}{6}\right]$
(s)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^3}{8} \end{bmatrix}$	$\left[\frac{1}{26},\ \frac{2}{13},\ \frac{2}{13},\ \frac{2}{13},\ \frac{2}{13},\ \frac{2}{13},\ \frac{2}{13},\ \frac{1}{26}\right]^T$
(t)	$\begin{bmatrix} 1 & -\frac{h}{2} & -\frac{h}{2} & -\frac{h}{2} & \frac{h^2}{4} & \frac{h^2}{4} & \frac{h^2}{4} & -\frac{h^3}{8} \\ 1 & \frac{h}{4} & -\frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & -\frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^3}{64} \\ 1 & \frac{h}{4} & -\frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & -\frac{h}{4} & \frac{h}{4} & \frac{h}{4} & -\frac{h^2}{16} & -\frac{h^2}{16} & \frac{h^2}{16} & -\frac{h^3}{64} \\ 1 & \frac{h}{4} & \frac{h}{4} & \frac{h}{4} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^2}{16} & \frac{h^3}{64} \end{bmatrix}$	$\left[\frac{1}{27},\ \frac{1}{6},\ \frac{1}{6},\ \frac{1}{9},\ \frac{1}{6},\ \frac{1}{9},\ \frac{1}{9},\ \frac{7}{54}\right]^T$

Table 5: Vandermonde matrix V and weights vector for each case.