Pacific Graphics 2018
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(Guest Editors)

Volume 37 (2018), Number 7

Supplementary Material: Local and Hierarchical Refinement for Subdivision Gradient Meshes

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In this document we give proofs for two important results from the paper and provide some additional results of the methods.

1. Proof of Lemma 1

Here we prove Lemma 1. The proof is based on a description of a diagrammatic representation of face topologies.

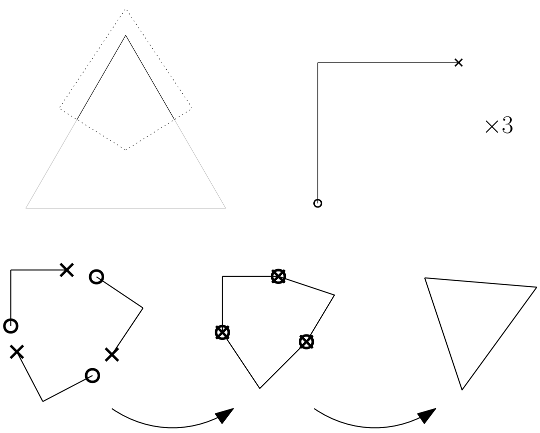

Figure 1: *We consider what is essentially only $\frac{1}{n}$-th of the original topology due to the n-fold rotational symmetry of C and T.* ***Top left:*** *A subsection of a triangle to be considered.* ***Top right:*** *A schematic representation of the triangle due to its rotational symmetry.* ***Bottom:*** *Reconstruction of the triangle from its schematic representation.*

We construct and compare the topologies generated by the operations $T \circ C$ and $C \circ T$ for a face of valency $n \geq 3$. Both T and C are invariant under cyclic reindexing of control points of such a face, so we need to consider only one corner of the original face; see Figure 1. To facilitate the diagrammatic proof, we invent a schematic representation for topologies that can informally be described as being invariant under rotation by $\frac{2\pi}{n}$.

In Figure 1, we show only what logically corresponds to $\frac{1}{n}$-th of the topology under consideration, and call this the *atomic topology*. The cross and circle denote how copies of the atomic topology should be *glued* together to create the full topology. Each cross matches with a circle.

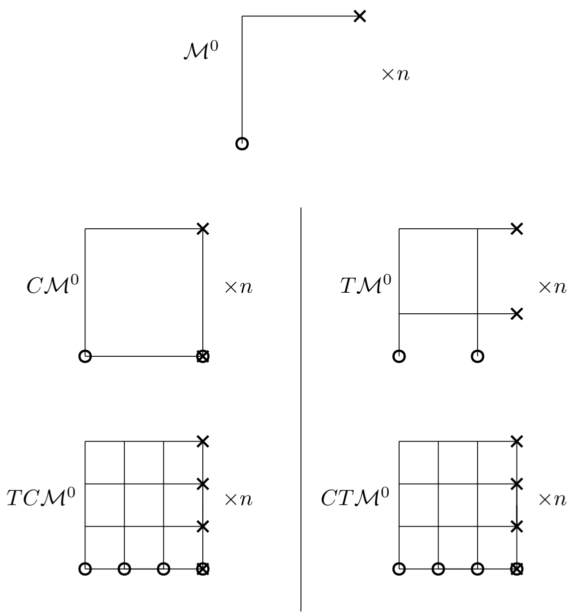

Figure 2: *A diagrammatic proof of Lemma 1. On the left side we construct the atomic topology of $TC\mathcal{M}^0$ and on the right side that of $CT\mathcal{M}^0$. The bottom row then shows that the two arising topologies are equivalent for any valency $n \geq 3$.*

Proof A diagrammatic proof of Lemma 1 is given in Figure 2. We construct the atomic topologies of both $TC\mathcal{M}^0$ and $CT\mathcal{M}^0$, and note by inspection that they are equal. □

2. Proof of Lemma 5

We now prove Lemma 5. We use the notation detailed in Section 5.3 of the main paper.

Proof If we subdivide up to some level q, the refinement of geometry as given in Equation (7) of the main paper can be written as

$$\begin{aligned}\tilde{\mathbf{x}}_q &= \left(\prod_{i=1}^{q} S_i\right)\mathbf{x}_0 + \mathbf{p}^{\text{ext}}_{q-k}\mathbf{c}_{\mathbf{x}},\\ &= \left(\prod_{i=1}^{q} S_i\right)\mathbf{x}_0 + \left(\prod_{j=k+1}^{q} S_j\right)\mathbf{p}^{\text{ext}}_{0}\mathbf{c}_{\mathbf{x}}\\ &= \left(\prod_{j=k+1}^{q} S_j\right)(\mathbf{x}_k + \mathbf{p}^{\text{ext}}_{0}\mathbf{c}_{\mathbf{x}}),\end{aligned}$$

where $\mathbf{x}_0$ contains the positions of the vertices in $\mathcal{M}^0 = T\widetilde{\mathcal{M}}$. □

A note on the above proof: The above holds for all $q \geq k \geq 0$, and therefore it also holds in the limiting case where $q \to \infty$ as all limit surfaces involved are well-defined.