Additional Material

Gradient-based steering for vision-based crowd simulation algorithms

T. B. Dutra[1], R. Marques[2,3], J. B. Cavalcante-Neto[1], C. A. Vidal[1] and J. Pettré[2]

[1]Universidade Federal do Ceará, Brazil

[2]INRIA Rennes - Bretagne Atlantique, France

[3]Universitat Pompeu Fabra, Spain

A Time to closest approach and distance at closest approach

Let us assume that the agents and the obstacles have a linear motion. Then $\mathbf{P}_a(t)$ and $\mathbf{P}_{o_i}(t)$ can be defined as:

$$\mathbf{P}_a(t) = \mathbf{p}_a + t\,\mathbf{v}_a\,, \quad (1)$$

$$\mathbf{P}_{o_i}(t) = \mathbf{p}_{o_i} + t\,\mathbf{v}_{o_i}\,, \quad (2)$$

where $\mathbf{P}_a(t)$ is the position of agent a and $\mathbf{P}_{o_i}(t)$ is the position of obstacle o_i, both at time t. The squared distance D^2 between agent a and obstacle o_i at time t is given by:

$$\begin{aligned} D^2(t) &= \|\mathbf{p}_{o_i} - \mathbf{p}_a + t\,(\mathbf{v}_{o_i} - \mathbf{v}_a)\,\|^2 \quad (3)\\ &= \|\mathbf{p}_{o_i|a} + t\,\mathbf{v}_{o_i|a}\|^2\,. \quad (4)\end{aligned}$$

The time to closest approach ($ttca_{o_i,a}$) between agent a and obstacle o_i is given by the following equation:

$$\frac{d}{dt}D^2(t) = 0, \quad (5)$$

where

$$\frac{d}{dt}D^2(t) = 2\left(\mathbf{p}_{o_i|a} + t\,\mathbf{v}_{o_i|a}\right)\cdot\mathbf{v}_{o_i|a}\,. \quad (6)$$

By solving Eq. (5) for t, we have that $ttca$ is given by:

$$ttca_{o_i,a} = \begin{cases} t \in \mathbb{R} & : \mathbf{v}_{o_i|a} = (0,0) \\ -\frac{\mathbf{p}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}{\|\mathbf{v}_{o_i|a}\|^2} & : \mathbf{v}_{o_i|a} \neq (0,0) \end{cases} \quad (7)$$

Once the value of $ttca_{o_i,a}$ is known, the $dca_{o_i,a}$ can be easily computed:

$$\begin{aligned} dca_{o_i,a} &= \sqrt{D^2(ttca_{o_i,a})} \quad (8)\\ &= \|\mathbf{p}_{o_i|a} + ttca_{o_i,a}\mathbf{v}_{o_i|a}\|, \quad (9)\end{aligned}$$

given that $\mathbf{x}^2 = \mathbf{x}\cdot\mathbf{x} = \|\mathbf{x}\|^2$.

B Gradient of the cost function

Computing the gradient

$$\nabla C_t = \frac{\partial C_t}{\partial s_a}.ds_a + \frac{\partial C_t}{\partial \theta_a}.d\theta_a$$

implies computing the partial derivatives given by:

$$\frac{\partial C_t}{\partial s_a} = \frac{\partial C_m}{\partial s_a} + \frac{\partial C_o}{\partial s_a} \quad (10)$$

and

$$\frac{\partial C_t}{\partial \theta_a} = \frac{\partial C_m}{\partial \theta_a} + \frac{\partial C_o}{\partial \theta_a}, \quad (11)$$

where

$$\frac{\partial C_o}{\partial s_a} = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial C_{o_i,a}}{\partial s_a} \quad (12)$$

and

$$\frac{\partial C_o}{\partial \theta_a} = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial C_{o_i,a}}{\partial \theta_a}\,. \quad (13)$$

The value of the partial derivatives of C_m (see Eqs. (10) and (11)), given by:

$$\frac{\partial C_m}{\partial s_a} = \frac{\Delta_s}{2\sigma_s^2}\,\exp\left(-\frac{1}{2}\left(\frac{\Delta_s}{\sigma_s}\right)^2\right) \quad (14)$$

and

$$\frac{\partial C_m}{\partial \theta_a} = -\frac{\alpha_g}{2\sigma_{\alpha_g}^2}\,\exp\left(-\frac{1}{2}\left(\frac{\alpha_g}{\sigma_{\alpha_g}}\right)^2\right)\,. \quad (15)$$

To determine the value the partial derivatives of C_o (see Eqs. (12) and (13)) the following quantities must to be computed:

$$\begin{aligned}\frac{\partial C_{o_i,a}}{\partial s_a} =& -C_{o_i,a}\left(\frac{\partial ttca_{o_i,a}}{\partial s_a}\frac{ttca_{o_i,a}}{\sigma_{ttca}^2}\right)\\ & -C_{o_i,a}\left(\frac{\partial dca_{o_i,a}}{\partial s_a}\frac{dca_{o_i,a}}{\sigma_{dca}^2}\right) \quad (16)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial C_{o_i,a}}{\partial \theta_a} =& -C_{o_i,a}\left(\frac{\partial ttca_{o_i,a}}{\partial \theta_a}\frac{ttca_{o_i,a}}{\sigma_{ttca}^2}\right)\\ & -C_{o_i,a}\left(\frac{\partial dca_{o_i,a}}{\partial \theta_a}\frac{dca_{o_i,a}}{\sigma_{dca}^2}\right)\,. \quad (17)\end{aligned}$$

In the next two sections of this appendix, we show how to compute the partial derivatives of $ttca_{o_i,a}$ and $dca_{o_i,a}$ required to evaluate Eqs. (16) and (17).

C Partial derivatives of $\mathtt{ttca}_{o_i,a}$

Let us assume that $\mathbf{v}_{o_i|a} \neq (0,0)$ in which case the $ttca_{o_i,a}$ is given by:

$$ttca_{o_i,a} = -\frac{f}{g} \quad (18)$$

where

$$f = \mathbf{p}_{o_i|a}\cdot\mathbf{v}_{o_i|a} \quad (19)$$

$$g = \mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\,. \quad (20)$$

The partial derivative of $ttca$ with respect to a hypothetical argument x is thus:

$$\frac{\partial ttca_{o_i,a}}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{g} + \frac{\frac{\partial g}{\partial x}\,f}{g^2}\ . \tag{21}$$

Let us recall that:

$$\mathbf{v}_{o_i|a} = \left(v_{xo_i} - s_a\cos\theta_a,\, v_{yo_i} - s_a\sin\theta_a\right)\ . \tag{22}$$

Let us also notice the following equalities:

$$\frac{\partial\,\mathbf{v}_{o_i|a}}{\partial\theta_a} = s_a\left(\sin\theta_a, -\cos\theta_a\right) = \left(v_{ya}, -v_{xa}\right) \tag{23}$$

and

$$\frac{\partial\,\mathbf{v}_{o_i|a}}{\partial s_a} = -(\cos\theta_a, \sin\theta_a) = -\hat{\mathbf{v}}_a\ . \tag{24}$$

To compute $\frac{\partial ttca_{o_i,a}}{\partial\theta_a}$ and $\frac{\partial ttca_{o_i,a}}{\partial s_a}$, we need thus to derive $\frac{\partial f}{\partial\theta_a}$, $\frac{\partial f}{\partial s_a}$, $\frac{\partial g}{\partial\theta_a}$ and $\frac{\partial g}{\partial s_a}$. Follow the two partial derivatives of f:

$$\begin{aligned}\frac{\partial f}{\partial\theta_a} &= \frac{\partial\left(\mathbf{p}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)}{\partial\theta_a} = \mathbf{p}_{o_i|a}\cdot\frac{\partial\,\mathbf{v}_{o_i|a}}{\partial\theta_a}\\ &= \mathbf{p}_{o_i|a}\cdot\left(v_{ya}, -v_{xa}\right)\end{aligned} \tag{25}$$

and

$$\begin{aligned}\frac{\partial f}{\partial s_a} &= \frac{\partial\left(\mathbf{p}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)}{\partial s_a} = \mathbf{p}_{o_i|a}\cdot\frac{\partial\,\mathbf{v}_{o_i|a}}{\partial s_a}\\ &= -\mathbf{p}_{o_i|a}\cdot\hat{\mathbf{v}}_a\ .\end{aligned} \tag{26}$$

The partial derivatives of g are given by:

$$\begin{aligned}\frac{\partial g}{\partial\theta_a} &= \frac{\partial\left(\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)}{\partial\theta_a}\\ &= \frac{\partial\mathbf{v}_{o_i|a}}{\partial\theta_a}\cdot\mathbf{v}_{o_i|a} + \mathbf{v}_{o_i|a}\cdot\frac{\partial\mathbf{v}_{o_i|a}}{\partial\theta_a}\\ &= \left(v_{ya}, -v_{xa}\right)\cdot\mathbf{v}_{o_i|a} + \mathbf{v}_{o_i|a}\cdot\left(v_{ya}, -v_{xa}\right)\\ &= 2\left(v_{ya}, -v_{xa}\right)\cdot\mathbf{v}_{o_i|a}\end{aligned} \tag{27}$$

and

$$\begin{aligned}\frac{\partial g}{\partial s_a} &= \frac{\partial\left(\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)}{\partial s_a}\\ &= \frac{\partial\mathbf{v}_{o_i|a}}{\partial s_a}\cdot\mathbf{v}_{o_i|a} + \mathbf{v}_{o_i|a}\cdot\frac{\partial\mathbf{v}_{o_i|a}}{\partial s_a}\\ &= -\hat{\mathbf{v}}_a\cdot\mathbf{v}_{o_i|a} - \mathbf{v}_{o_i|a}\cdot\hat{\mathbf{v}}_a\\ &= -2\,\hat{\mathbf{v}}_a\cdot\mathbf{v}_{o_i|a}\ .\end{aligned} \tag{28}$$

Using Eqs. (21), (25) and (27) we can compute:

$$\begin{aligned}\frac{\partial ttca_{o_i,a}}{\theta_a} &= -\frac{\mathbf{p}_{o_i|a}\cdot\left(v_{ya}, -v_{xa}\right)}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}\\ &\quad + \frac{\left(2\left(v_{ya}, -v_{xa}\right)\cdot\mathbf{v}_{o_i|a}\right)\left(\mathbf{p}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)}{\left(\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)^2}\\ &= -\frac{\mathbf{p}_{o_i|a}\cdot\left(v_{ya}, -v_{xa}\right)}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}\\ &\quad + \frac{2\,ttca_{o_i,a}\,\mathbf{v}_{o_i|a}\cdot\left(v_{ya}, -v_{xa}\right)}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}\\ &= -\frac{\left(\mathbf{p}_{o_i|a} + 2\,ttca_{o_i,a}\,\mathbf{v}_{o_i|a}\right)\cdot\left(v_{ya}, -v_{xa}\right)}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}\ .\end{aligned} \tag{29}$$

Using Eqs. (21), (26) and (28) we can compute:

$$\begin{aligned}\frac{\partial ttca_{o_i,a}}{s_a} &= \frac{\mathbf{p}_{o_i|a}\cdot\hat{\mathbf{v}}_a}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}} - \frac{2\left(\hat{\mathbf{v}}_a\cdot\mathbf{v}_{o_i|a}\right)\left(\mathbf{p}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)}{\left(\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}\right)^2}\\ &= \frac{\left(\mathbf{p}_{o_i|a}\cdot\hat{\mathbf{v}}_a\right) + \left(2\,ttca_{o_i,a}\,\mathbf{v}_{o_i|a}\cdot\hat{\mathbf{v}}_a\right)}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}\\ &= \frac{\left(\mathbf{p}_{o_i|a} + 2\,ttca_{o_i,a}\,\mathbf{v}_{o_i|a}\right)\cdot\hat{\mathbf{v}}_a}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}\ .\end{aligned} \tag{30}$$

D Partial derivatives of $\mathrm{dca}_{o_i,a}$

Let us recall the expression of $dca_{o_i,a}$:

$$\begin{aligned}dca_{o_i,a} &= \|\mathbf{dca}_{o_i,a}\| = \|\mathbf{p}_{o_i|a} + ttca_{o_i,a}\mathbf{v}_{o_i|a}\|\\ &= \left(\mathbf{dca}_{o_i,a}\cdot\mathbf{dca}_{o_i,a}\right)^{1/2}\ .\end{aligned} \tag{31}$$

The partial derivative of $dca_{o_i,a}$ with respect to a hypothetical argument x is given by:

$$\begin{aligned}\frac{\partial dca_{o_i,a}}{\partial x} &= \frac{1}{2}\left(\mathbf{dca}_{o_i,a}\cdot\mathbf{dca}_{o_i,a}\right)^{-1/2}\frac{\partial\left(\mathbf{dca}_{o_i,a}\cdot\mathbf{dca}_{o_i,a}\right)}{\partial x}\\ &= \frac{1}{2\,dca_{o_i,a}}\frac{\partial\left(\mathbf{dca}_{o_i,a}\cdot\mathbf{dca}_{o_i,a}\right)}{\partial x}\\ &= \frac{1}{2\,dca_{o_i,a}}\frac{\partial\left(\|\mathbf{dca}_{o_i,a}\|^2\right)}{\partial x}\ .\end{aligned} \tag{32}$$

The rightmost term of Eq. (32) can be developed as:

$$\begin{aligned}\frac{\partial\left(\|\mathbf{dca}_{o_i,a}\|^2\right)}{\partial x} &= \frac{\partial\mathbf{dca}_{o_i,a}}{\partial x}\cdot\mathbf{dca}_{o_i,a}\\ &\quad + \frac{\partial\mathbf{dca}_{o_i,a}}{\partial x}\cdot\mathbf{dca}_{o_i,a}\\ &= 2\,\frac{\partial\mathbf{dca}_{o_i,a}}{\partial x}\cdot\mathbf{dca}_{o_i,a}\\ &= 2\,\frac{\partial\left(\mathbf{p}_{o_i|a} + ttca_{o_i,a}\mathbf{v}_{o_i|a}\right)}{\partial x}\cdot\mathbf{dca}_{o_i,a}\end{aligned} \tag{33}$$

given that $\mathbf{p}_{o_i|a}$ is a constant, this equation can be simplified as:

$$\frac{\partial\left(\|\mathbf{dca}_{o_i,a}\|^2\right)}{\partial x} = 2\left(\frac{\partial(ttca_{o_i,a}\mathbf{v}_{o_i|a})}{\partial x}\right)\cdot\mathbf{dca}_{o_i,a} \tag{34}$$

where

$$\frac{\partial(ttca_{o_i,a}\mathbf{v}_{o_i|a})}{\partial x} = \frac{\partial ttca_{o_i,a}}{\partial x}\mathbf{v}_{o_i|a} + ttca_{o_i,a}\frac{\partial\mathbf{v}_{o_i|a}}{\partial x}\ . \tag{35}$$

To compute the partial derivative of $dca_{o_i,a}$ with respect to θ_a, we can use the results from Eqs. (23), (29), (32) and (34), yielding:

$$\begin{aligned}\frac{\partial dca_{o_i,a}}{\partial\theta_a} &= \left(-\frac{\left(\mathbf{p}_{o_i|a} + 2\,ttca_{o_i,a}\,\mathbf{v}_{o_i|a}\right)\cdot\left(v_{ya}, -v_{xa}\right)}{\mathbf{v}_{o_i|a}\cdot\mathbf{v}_{o_i|a}}\mathbf{v}_{o_i|a}\right.\\ &\quad\left. + ttca_{o_i,a}(v_{ya}, -v_{xa})\right)\cdot\mathbf{dca}_{o_i,a}\frac{1}{dca_{o_i,a}}\\ &= \frac{\mathbf{dca}_{o_i,a}\cdot\left(\frac{\partial ttca_{o_i,a}}{\partial\theta_a}\mathbf{v}_{o_i|a} + ttca_{o_i,a}\left(v_{ya}, -v_{xa}\right)\right)}{dca_{o_i,a}}\ .\end{aligned} \tag{36}$$

Similarly, to compute the partial derivative of $dca_{o_i,a}$ with respect to s_a, we can use the results from Eqs. (24), (30), (32) and (34).

$$\frac{\partial dca_{o_i,a}}{\partial s_a} = \left(\frac{\left(\mathbf{p}_{o_i|a} + 2\, ttca_{o_i,a}\, \mathbf{v}_{o_i|a}\right) \cdot (v_{ya}, -v_{xa})}{\mathbf{v}_{o_i|a} \cdot \mathbf{v}_{o_i|a}} \mathbf{v}_{o_i|a} - ttca_{o_i,a} \hat{\mathbf{v}}_a \right) \cdot \mathbf{dca}_{o_i,a} \frac{1}{dca_{o_i,a}}$$

$$= \frac{\mathbf{dca}_{o_i,a} \cdot \left(\frac{\partial ttca_{o_i,a}}{\partial s} \mathbf{v}_{o_i|a} - ttca_{o_i,a} \hat{\mathbf{v}}_a \right)}{dca_{o_i,a}} . \quad (37)$$

E Dependence on Camera Resolution

Synthetic vision algorithms manipulate large amounts of data (12288 pixels per agent for a 256×48 camera resolution, for example). The main bottleneck is the texture download from GPU to CPU. This can be alleviated by downsizing the camera's resolution. Fig. 1 shows the effect of resolution decrease on simulation results for our model and for OSV. Results show that our new technique is much less sensitive to the camera resolution parameter. Visual differences in the trajectories can hardly be observed even when reducing the original resolution by 93.75%. A special case of camera resolution is 256×1, in which case the agent keeps a wide view (rightmost column of Fig. 1). Finally, Fig. 2 shows the performance of our model when varying the number of agents and the camera resolution. As expected, the performance decreases as the number of agents increases.

F Complete results

In this section we provide the complete set of results used for the article, as well as some variations of the presented scenarios.

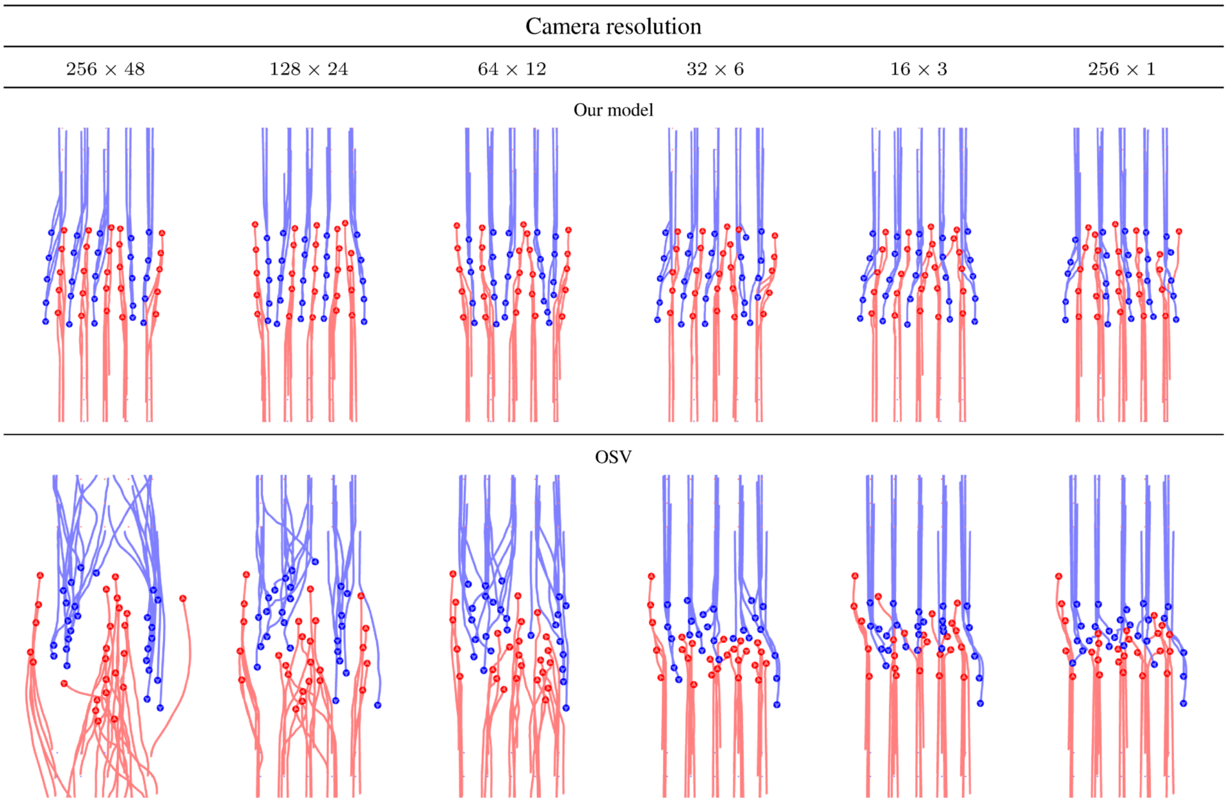


Figure 1: *Impact of the camera resolution for our model and OSV. In our model the obstacles perception and anticipation are only slightly affected, even using a 1D camera resolution.*

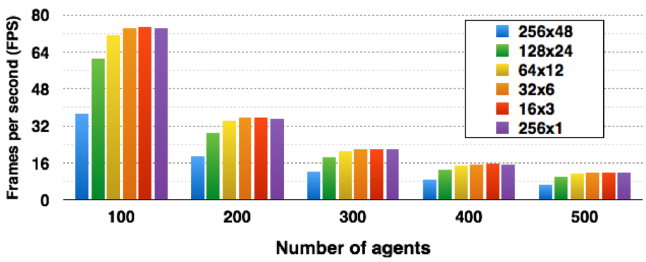


Figure 2: *Performance for the Opposite scenario varying the number of agents and the camera resolution.*

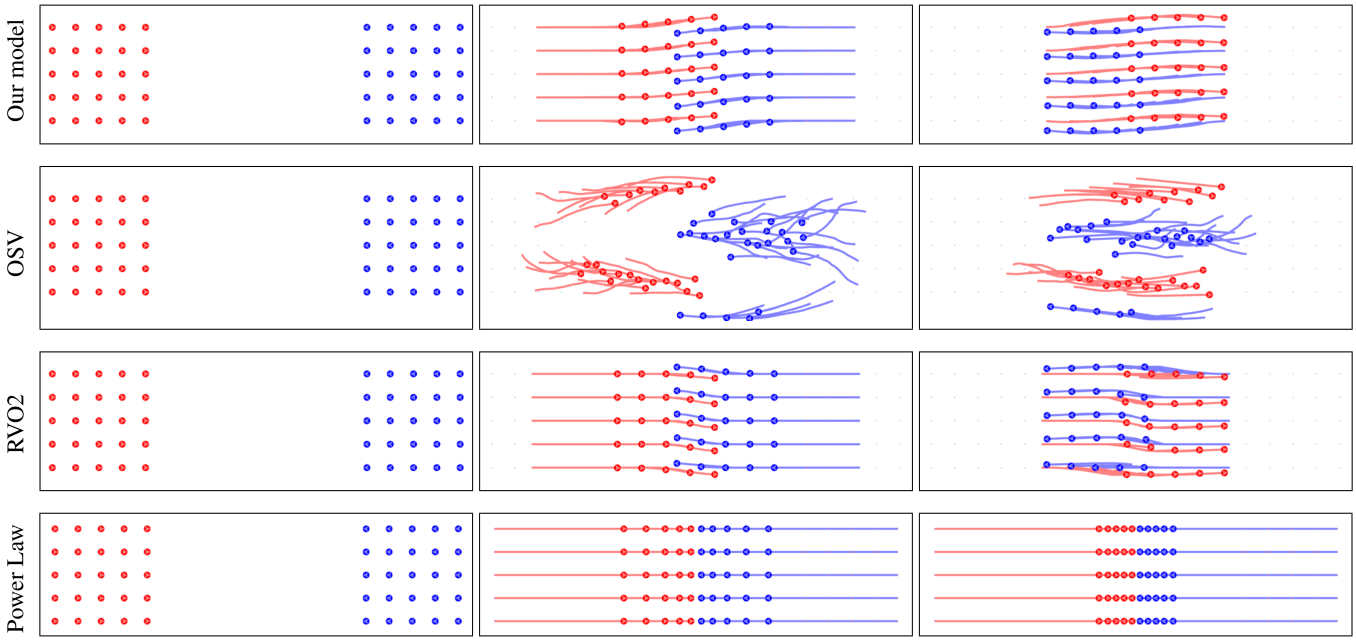


Figure 3: *Comparison of the results for the **Opposite scenario** with **structured initial positions**. The two groups of agents (red and blue) have as goal to switch positions. Results are shown for our model, OSV, RVO2 and Power Law.*

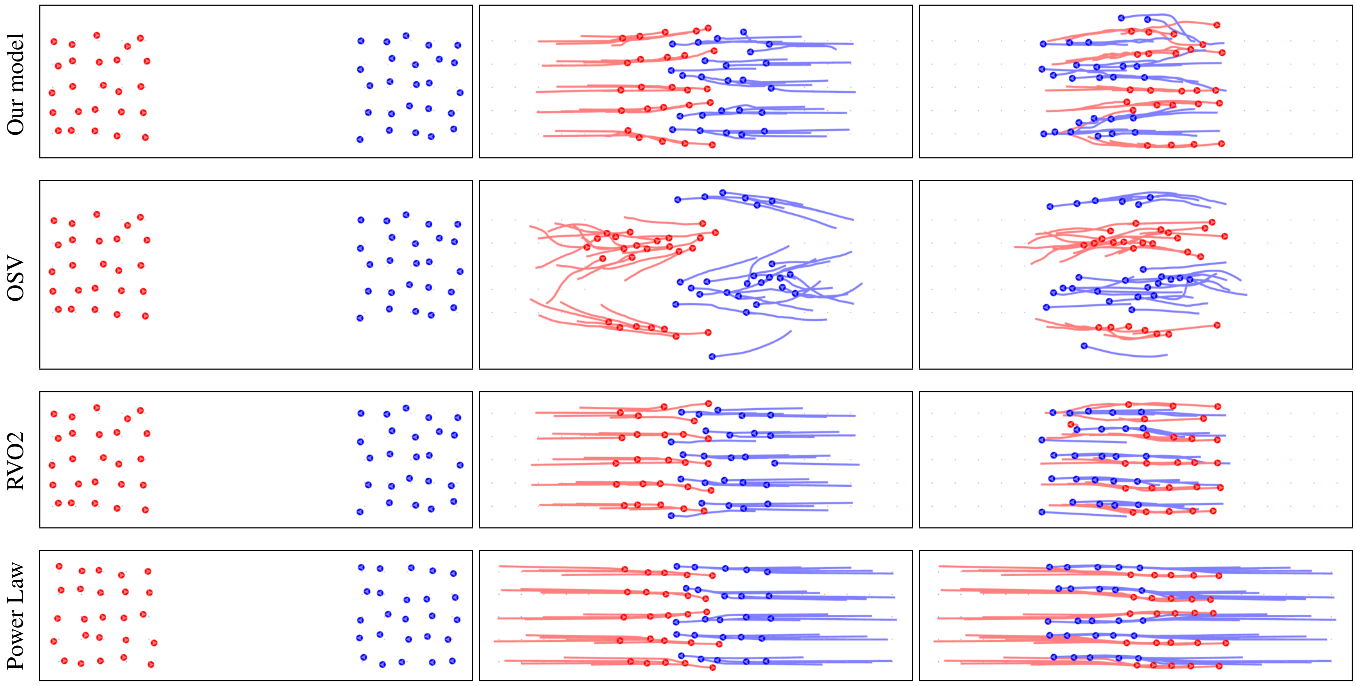


Figure 4: *Comparison of the results for the **Opposite scenario** with **noisy initial positions**. The two groups of agents (red and blue) have as goal to switch positions. Results are shown for our model, OSV, RVO2 and Power Law.*

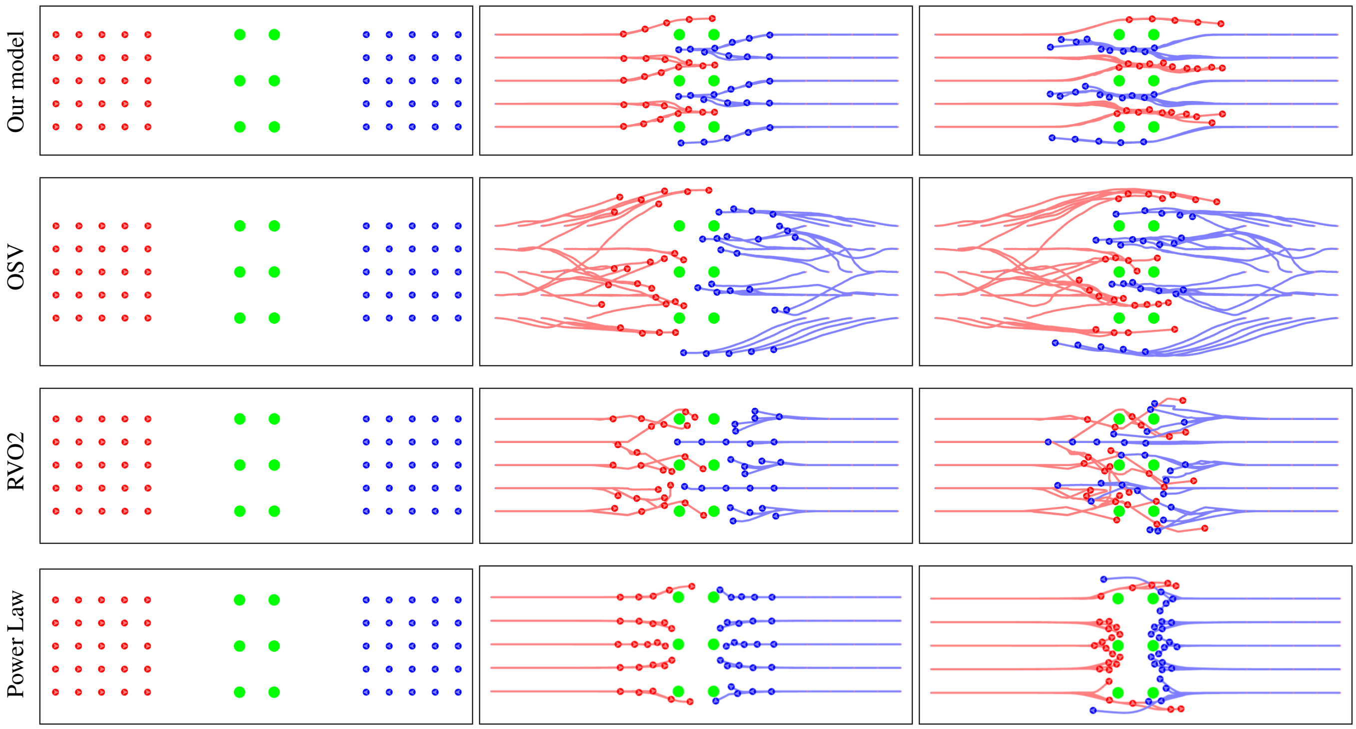


Figure 5: *Comparison of the results for the* ***Columns scenario****. The two groups of agents (red and blue) have as goal to switch positions. Results are shown for our model, OSV, RVO2 and Power Law.*

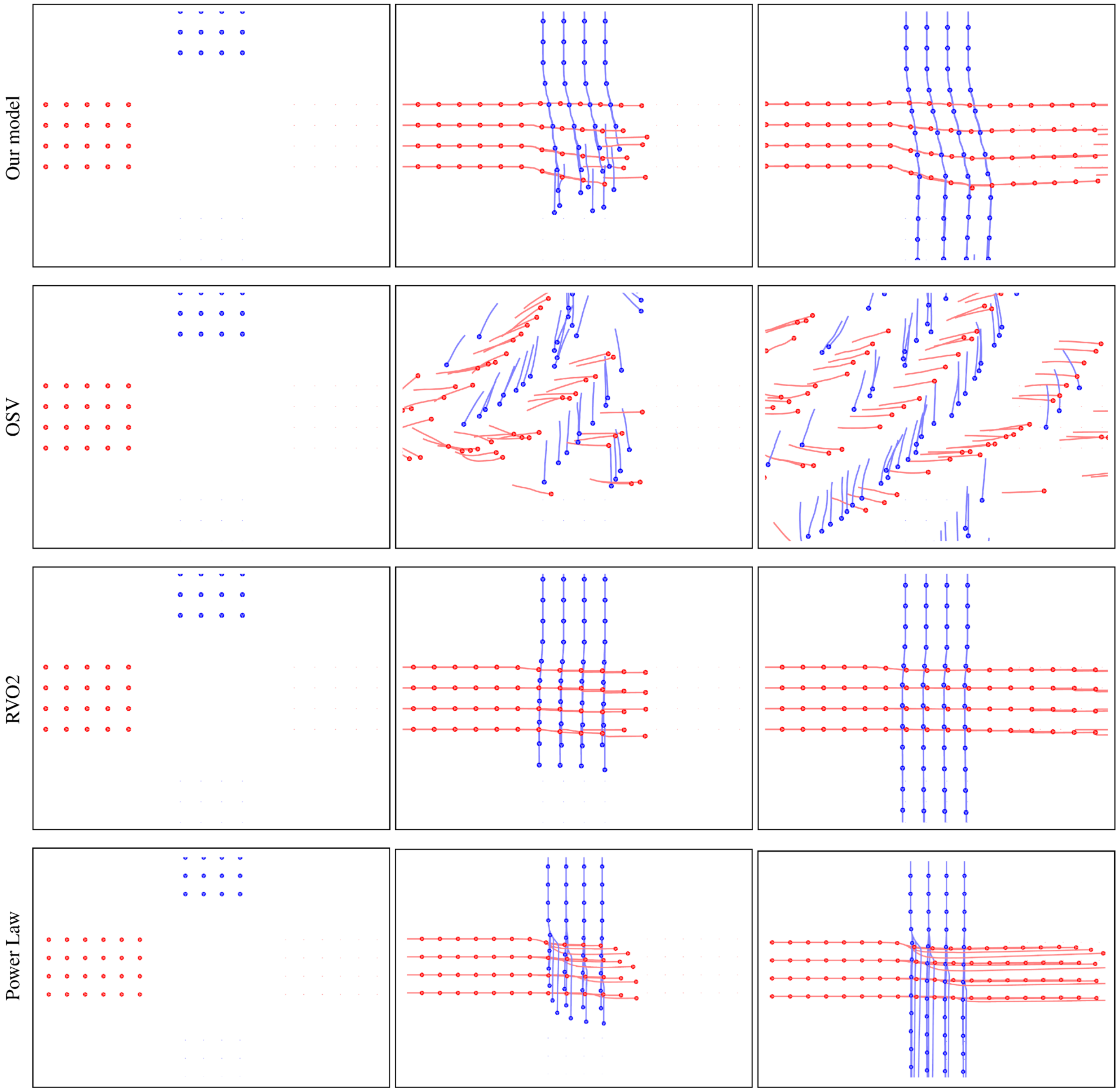


Figure 6: *Comparison of the results for the **Crossing scenario** with **many agents** and **structured initial positions**. The two groups of agents (red and blue) move perpendicularly and must cross each other to reach their goal. Results are shown for our model, OSV, RVO2 and Power Law.*

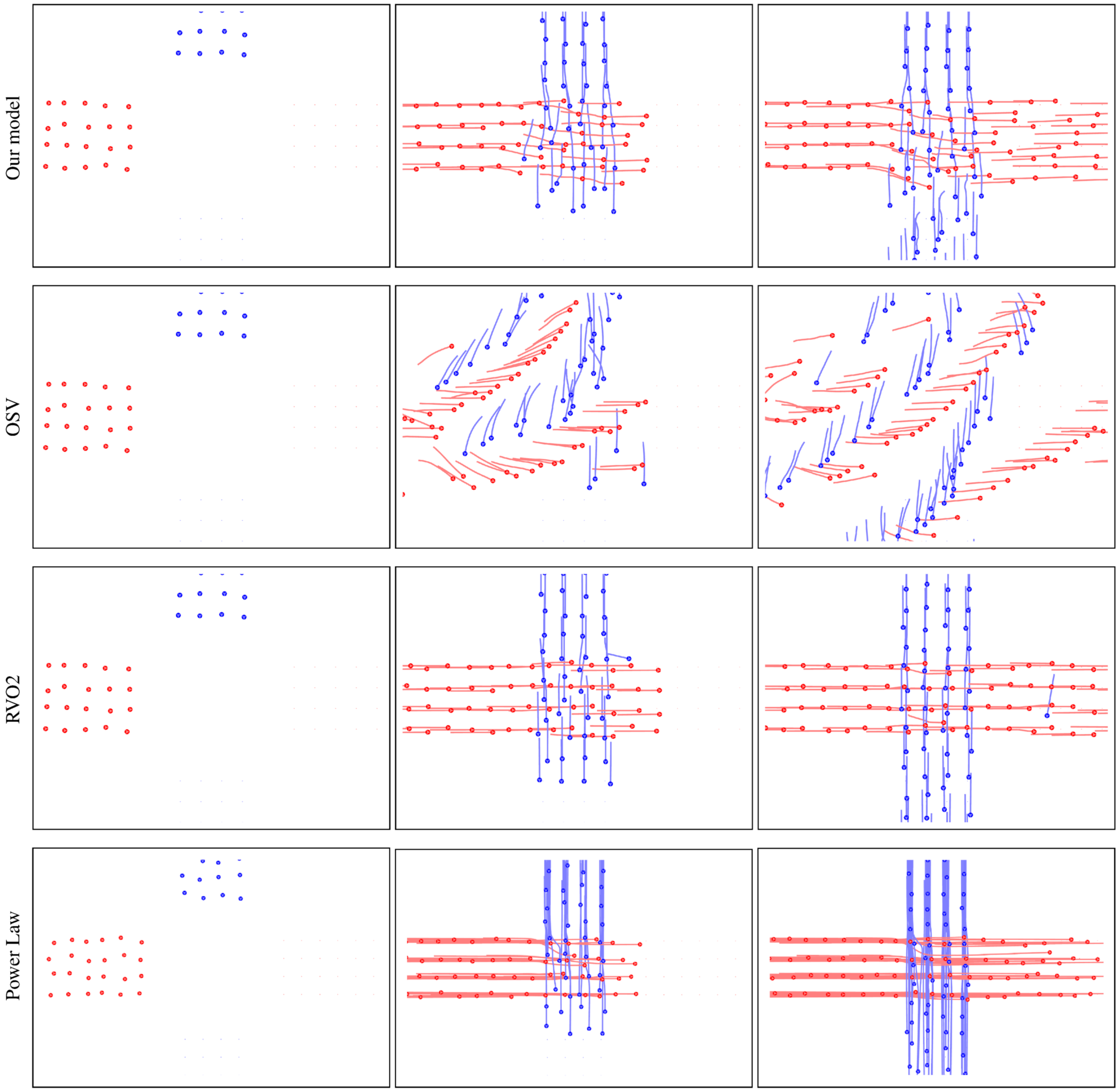


Figure 7: *Comparison of the results for the* ***Crossing scenario*** *with* ***many agents*** *and* ***noisy initial positions****. The two groups of agents (red and blue) move perpendicularly and must cross each other to reach their goal. Results are shown for our model, OSV, RVO2 and Power Law.*

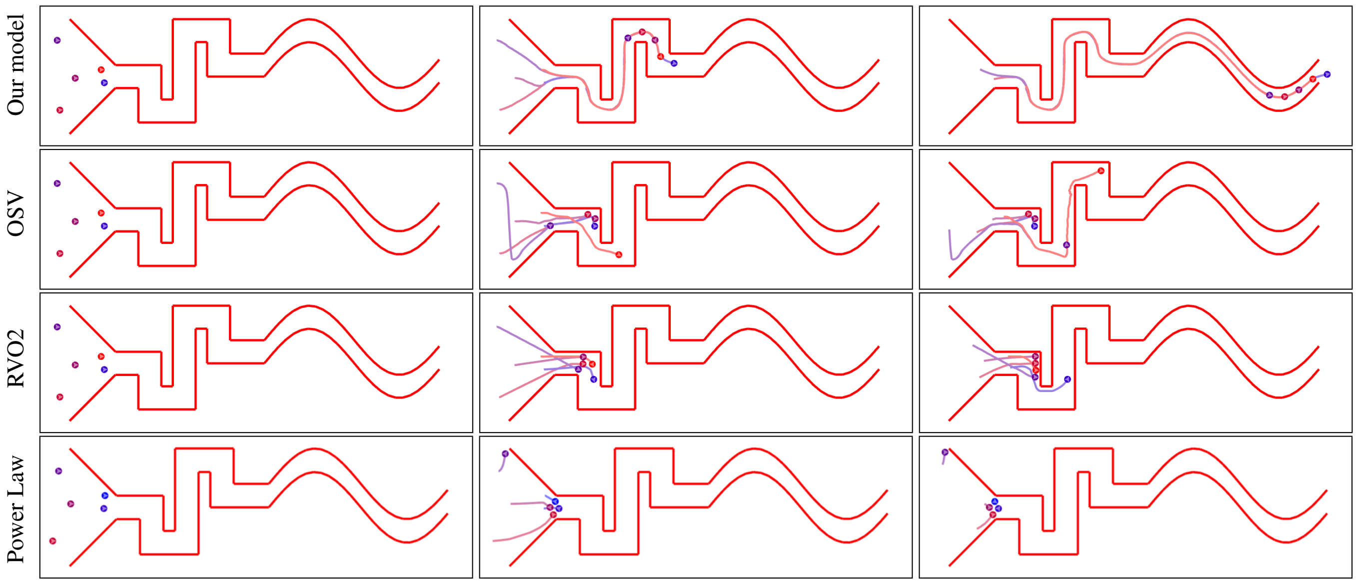


Figure 8: *Comparison of the results for the **S-Corridor scenario**. The agents must traverse the corridor so as to reach their goal. No global path planner is used. Results are shown for our model, OSV, RVO2 and Power Law.*

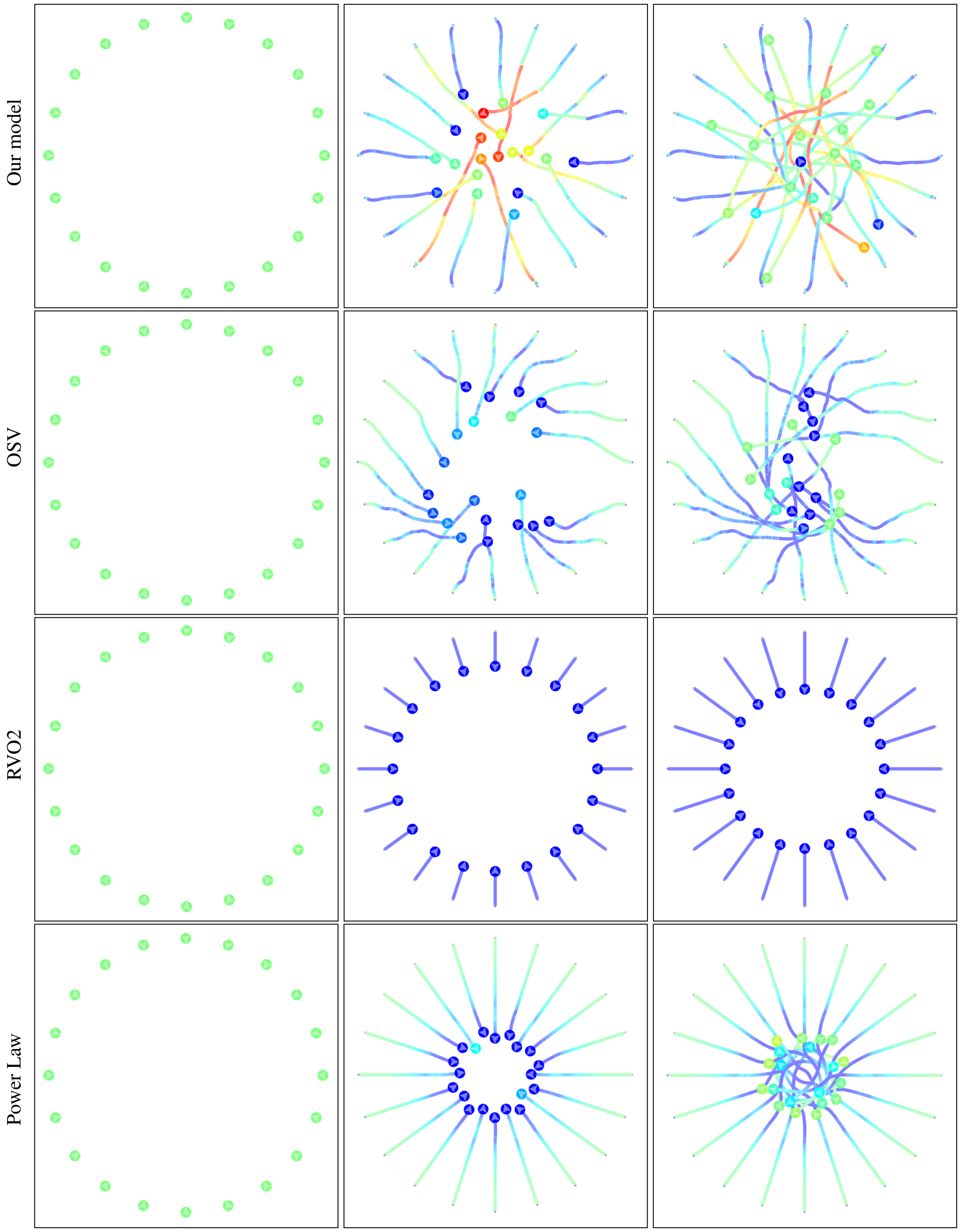


Figure 9: *Comparison of the results for the* ***Circle scenario*** *with* ***symmetric initial positions****. The goal of the agents is to reach the diametrically opposed position. The agents color encodes the speed: dark blue means the agent is stopped or moving slower than its comfort speed; light green means the agent is moving at its comfort speed; and red means the agent is moving faster than its comfort speed. Results are shown for our model, OSV, RVO2 and Power Law.*

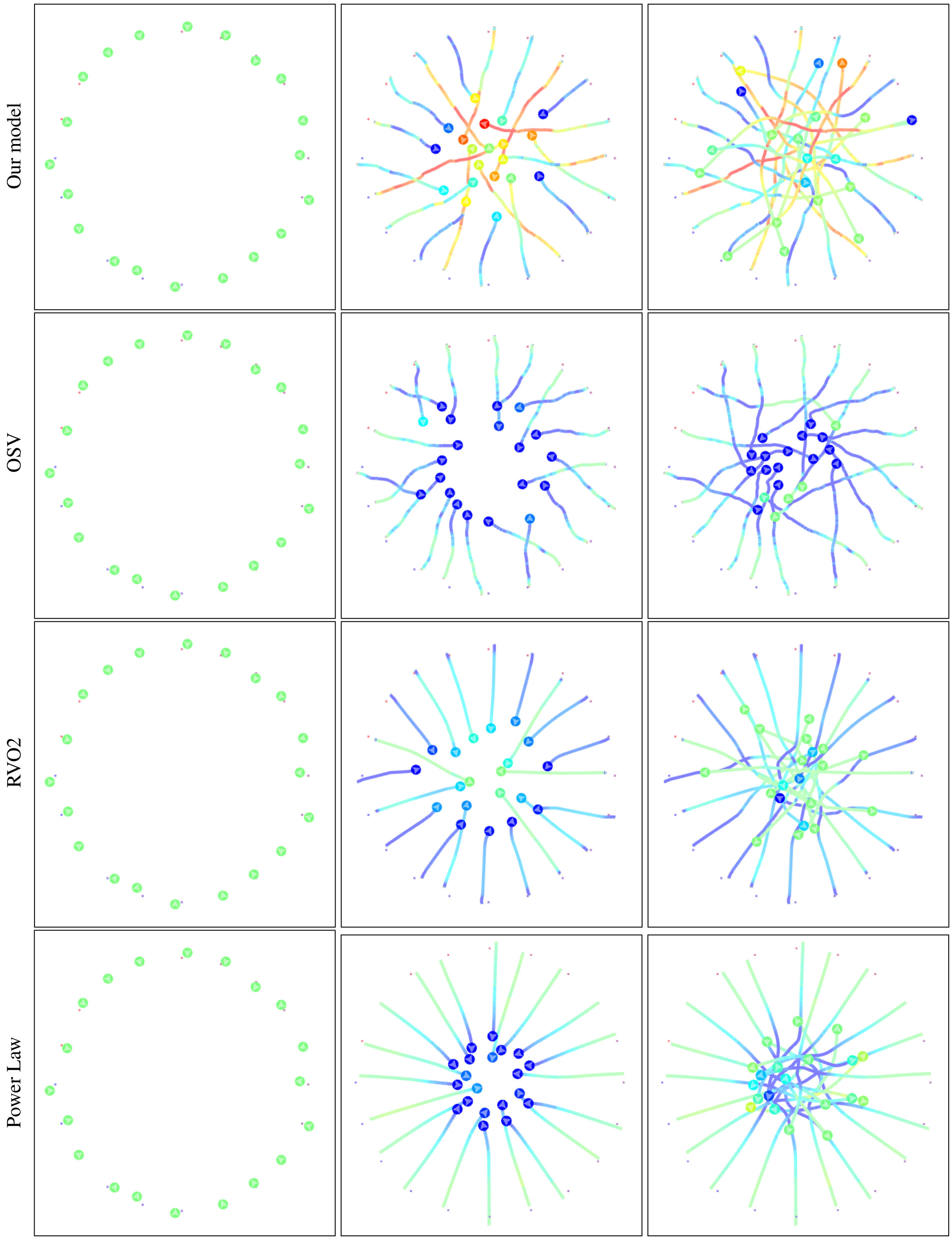
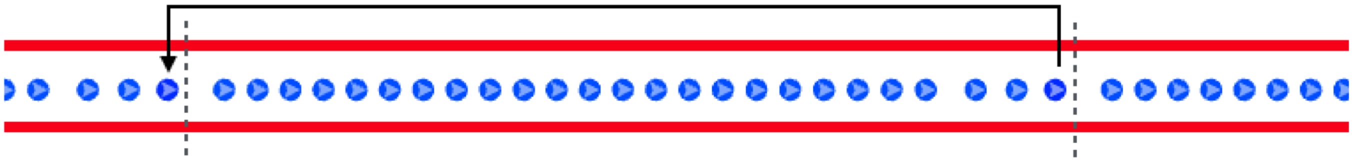
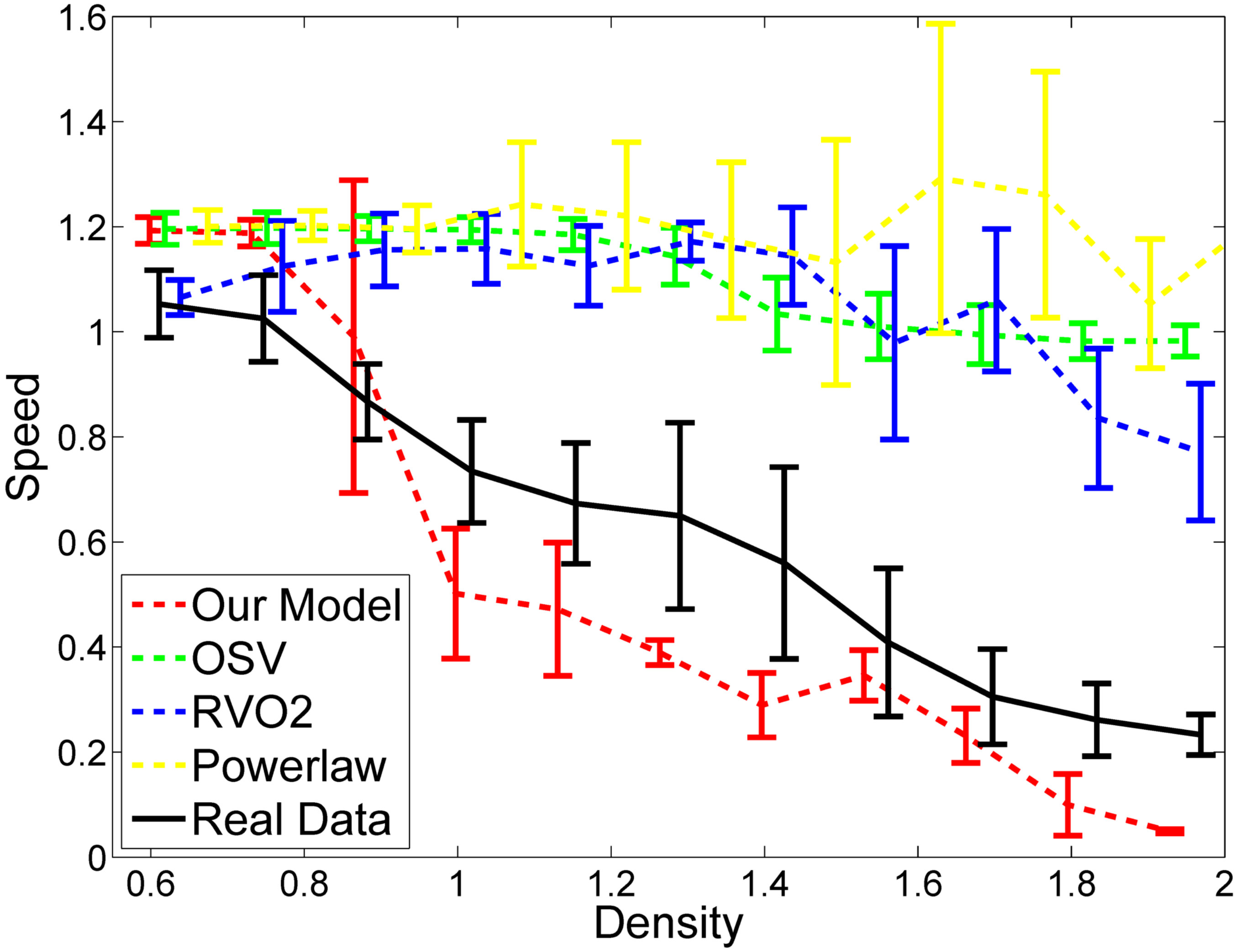


Figure 10: *Comparison of the results for the* ***Circle scenario*** *with* ***noisy initial positions****. The goal of the agents is to reach the diametrically opposed position. The agents color encodes the speed: dark blue means the agent is stopped or moving slower than its comfort speed; light green means the agent is moving at its comfort speed; and red means the agent is moving faster than its comfort speed. Results are shown for our model, OSV, RVO2 and Power Law.*



(a) *1-D Periodic Corrridor* scenario.



(b) Fundamental diagram.

Figure 11: *Results for the* 1-D Periodic Corridor *scenario. A way of quantifying global features of a model is to determine its fundamental diagram (FD), which measures the speed of the agents as a function of the density of agents in a given area. In the scenario shown in (a), we measured the agents' speed as a function of the density. The bottom image compares the fundamental diagram (relation between density and speed) of the four tested models and real data. Note that the curve corresponding to our model, in red, follows the general trends observed in the real data, as opposed to those of the other tested models.*